Aspects of strong coupled non-conformal gauge theories at finite temperature

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with: Chris Pagnutti, Andre Blanchard, Patrick Kernel, $\cdots$

Outline of the talk:

- Motivation
- CFT plasmas and why they can not answer the two questions raised in the motivation
- $\mathcal{N} = 2^*$ gauge theory as a toy model:

$\Longrightarrow$ Susy/non-susy mass deformations of $\mathcal{N} = 4$ in QFT/supergravity

$\Longrightarrow$ Thermodynamics of $\mathcal{N} = 2^*$ for (non-)susy mass-deformations

$\Longrightarrow$ Bulk viscosity of $\mathcal{N} = 2^*$ for (non)-susy mass-deformations

$\Longrightarrow$ "$\mathcal{N} = 2^*$-based" thermometer for RHIC

- Conclusions and future directions

RHIC experiment at BNL collides bunches of Au ions at 99.995% of the speed of light. The ions 'melt' and produce a new state of matter: the Quark-Gluon-Plasma (QGP). The typically temperature of the QGP is roughly estimated to be $1.5T_{deconfinement}$; thus the plasma is expected (and is observed) to be strongly coupled. We would like to study the properties of this QGP 'liquid'.

Q.1: How we can measure the temperature of the QGP ball?

Q.2: Hydrodynamic simulations of the QGP quite well agree with the experimental data; Kharzeev et.al proposed that fast equilibration is due to the large bulk viscosity of QGP near $T_{deconfinement}$. Harvey Meyer's lattice simulations suggests that $\frac{\zeta}{\eta} \sim 8 \cdots 10$ at $T = 1.02T_{deconfinement}$. Can we understand/see the grows of bulk viscosity from gauge/string duality?

In this talk we would like to answer **Q.1** and **Q.2**.

A.1:'Rajagopal's thermometer'. Assume that there is an ideal situation and we are able to extract as precise data as possible from the experiment to 'tune' our hydrodynamic codes. From the hydrodynamic codes we expect to obtain the jet quenching parameter $\hat{q}$ as a function of the speed of sound c_s, $\hat{q} = \hat{q}(c_s)$. Now, using the static lattice simulations we can relate

$$c_s^2 = \frac{\partial P}{\partial \epsilon} \qquad \Longrightarrow \qquad c_s^2 = c_s^2(T/T_d)$$

and thus obtain $\hat{q} = \hat{q}(T/T_d)$. We would like to use toy models of gauge/string duality to obtain

$$\frac{\hat{q}}{s} = \frac{\hat{q}}{s}\left(c_s^2\right)$$

Is our thermometer universal?

A.2: Use toy models of gauge/string duality to compute $\frac{\zeta}{\eta}$

Why a CFT plasma fails to answer **Q.1** and **Q.2**?

Q.1 In CFT at thermal equilibrium

$$T^{\mu}_{\mu} = 0 \qquad \Longrightarrow \qquad \epsilon = 3P \qquad \Longrightarrow \qquad c_s^2 = \frac{\partial P}{\partial \epsilon} = \frac{1}{3} = \text{constant!}$$

Q.2 For any fluid, to first order in velocity gradients,

$$T_{\mu\nu} = \epsilon\, u_\mu u_\nu + P\Delta_{\mu\nu} - \eta\sigma_{\mu\nu} - \zeta\Delta_{\mu\nu}\left(\nabla \cdot u\right)$$

where $\{\eta, \zeta\}$ are the shear and the bulk viscosity, $\{\Delta_{\mu\nu}, \sigma_{\mu\nu}\}$ are symmetric transverse tensor constructed from u_μ (in case of Δ) and $\nabla_\mu u_\nu$ (in case of σ); also

$$\Delta^{\mu}_{\mu} = 3, \qquad \sigma^{\mu}_{\mu} = 0$$

Here again, the tracelessness of $T_{\mu\nu}$ (as required for the unbroken scale invariance), implies

$$\zeta_{CFT} = 0$$

Introduce

$$\delta \equiv c_s^2 - \frac{1}{3}$$

a deviation from the conformality.

$$\delta \neq 0$$

due to

$\swarrow$ $\searrow$

$\left(\frac{m}{T}\right)$	$\left(\frac{\Lambda}{T}\right)$
explicit breaking	spontaneous breaking
by mass terms	by a strong coupling scale
$\Uparrow$	$\Uparrow$
mass deformed $\mathcal{N} = 4$	cascading gauge theory

In this talk we discuss in details explicit breaking of the scale invariance by mass terms.

$\mathcal{N} = 2^*$ gauge theory (a QFT story)

$\Longrightarrow$ Start with $\mathcal{N} = 4\ SU(N)$ SYM. In $\mathcal{N} = 1$ 4d susy language, it is a gauge theory of a vector multiplet V, an adjoint chiral superfield Φ (related by $\mathcal{N} = 2$ susy to V) and an adjoint pair $\{Q, \tilde{Q}\}$ of chiral multiplets, forming an $\mathcal{N} = 2$ hypermultiplet. The theory has a superpotential:

$$W = \frac{2\sqrt{2}}{g_{YM}^2} \mathrm{Tr} \left(\left[Q, \tilde{Q} \right] \Phi \right)$$

We can break susy down to $\mathcal{N} = 2$, by giving a mass for $\mathcal{N} = 2$ hypermultiplet:

$$W = \frac{2\sqrt{2}}{g_{YM}^2} \mathrm{Tr} \left(\left[Q, \tilde{Q} \right] \Phi \right) + \frac{m}{g_{YM}^2} \left(\mathrm{Tr} Q^2 + \mathrm{Tr} \tilde{Q}^2 \right)$$

This theory is known as $\mathcal{N} = 2^*$ gauge theory

When $m \neq 0$, the mass deformation lifts the $\{Q, \tilde{Q}\}$ hypermultiplet moduli directions; we are left with the $(N-1)$ complex dimensional Coulomb branch, parametrized by

$$\Phi = \mathrm{diag}\left(\mathrm{a_1}, \mathrm{a_2}, \cdots, \mathrm{a_N}\right) \,, \qquad \sum_{\mathrm{i}} \mathrm{a_i} = 0$$

We will study $\mathcal{N} = 2^*$ gauge theory at a particular point on the Coulomb branch moduli space:

$$a_i \in [-a_0, a_0] \,, \qquad a_0^2 = \frac{m^2 g_{YM}^2 N}{\pi}$$

with the (continuous in the large N-limit) linear number density

$$\rho(a) = \frac{2}{m^2 g_{YM}^2} \sqrt{a_0^2 - a^2} \,, \qquad \int_{-a_0}^{a_0} da \; \rho(a) = N$$

Reason: we understand the dual supergravity solution only at this point on the moduli space.

$\mathcal{N}=2^*$ gauge theory (a supergravity story — a.k.a Pilch-Warner flow)

Consider 5d gauged supergravity, dual to $\mathcal{N}=2^*$ gauge theory. The effective five-dimensional action is

$$S=\frac{1}{4\pi G_5}\int_{\mathcal{M}_5} d\xi^5\sqrt{-g}\left(\tfrac{1}{4}R-(\partial\alpha)^2-(\partial\chi)^2-\mathcal{P}\right)\ ,$$

where the potential $\mathcal{P}$ is

$$\mathcal{P}=\frac{1}{16}\left[\left(\frac{\partial W}{\partial\alpha}\right)^2+\left(\frac{\partial W}{\partial\chi}\right)^2\right]-\frac{1}{3}W^2\,,$$

with the superpotential

$$W=-\frac{1}{\rho^2}-\frac{1}{2}\rho^4\cosh(2\chi)\,,\qquad \alpha\equiv\sqrt{3}\ln\rho$$

$\Longrightarrow$ The 2 supergravity scalars $\{\alpha,\chi\}$ are holographic dual to dim-2 and dim-3 operators which are nothing but (correspondingly) the bosonic and the fermionic mass terms of the $\mathcal{N}=4\to\mathcal{N}=2$ SYM mass deformation.

PW geometry ansatz:

$$ds_5^2 = e^{2A}\left(-dt^2 + d\vec{x}^{\,2}\right) + dr^2$$

solving the Killing spinor equations, we find a susy flow:

$$\frac{dA}{dr} = -\frac{1}{3}W\,, \qquad \frac{d\alpha}{dr} = \frac{1}{4}\frac{\partial W}{\partial \alpha}\,, \qquad \frac{d\chi}{dr} = \frac{1}{4}\frac{\partial W}{\partial \chi}$$

Solutions to above are characterized by a single parameter k:

$$e^A = \frac{k\rho^2}{\sinh(2\chi)}\,, \qquad \rho^6 = \cosh(2\chi) + \sinh^2(2\chi)\,\ln\frac{\sinh(\chi)}{\cosh(\chi)}$$

In was found (Polchinski,Peet,AB) that

$$k = 2m$$

Introduce

$$\hat{x} \equiv e^{-r/2} \,,$$

then

$$\chi = k\hat{x}\Big[1 + k^2\hat{x}^2\left(\tfrac{1}{3} + \tfrac{4}{3}\ln(k\hat{x})\right) + k^4\hat{x}^4\left(-\tfrac{7}{90} + \tfrac{10}{3}\ln(k\hat{x}) + \tfrac{20}{9}\ln^2(k\hat{x})\right)$$

$$+\mathcal{O}\left(k^6\hat{x}^6\ln^3(k\hat{x})\right)\Big] \,,$$

$$\rho = 1 + k^2\hat{x}^2\left(\tfrac{1}{3} + \tfrac{2}{3}\ln(k\hat{x})\right) + k^4\hat{x}^4\left(\tfrac{1}{18} + 2\ln(k\hat{x}) + \tfrac{2}{3}\ln^2(k\hat{x})\right) + \mathcal{O}\left(k^6\hat{x}^6\ln^3(k\hat{x})\right) \,,$$

$$A = -\ln(2\hat{x}) - \tfrac{1}{3}k^2\hat{x}^2 - k^4\hat{x}^4\left(\tfrac{2}{9} + \tfrac{10}{9}\ln(k\hat{x}) + \tfrac{4}{9}\ln^2(k\hat{x})\right) + \mathcal{O}\left(k^6\hat{x}^6\ln^3(k\hat{x})\right)$$

Or in standard Poincare-patch AdS_5 radial coordinate:

$$A \propto \ln r, \qquad \alpha \propto \frac{k^2 \ln r}{r^2}\,, \qquad \chi \propto \frac{k}{r}\,, \qquad r \to \infty$$

$\Longrightarrow$ Notice that the nonnormalizable components of $\{\alpha, \chi\}$ are related — this is holographic dual to $\mathcal{N} = 2$ susy preserving condition on the gauge theory side:

$$m_b = m_f$$

But in general, we can keep $m_b \neq m_f$:

$$A \propto \ln r, \qquad \alpha \propto \frac{m_b^2 \ln r}{r^2}, \qquad \chi \propto \frac{m_f}{r}, \qquad r \to \infty$$

The precise relation, including numerical coefficients can be works out.

$\Longrightarrow$ There are no singularity-free flows (geometries) with $m_b \neq m_f$ and at zero temperature $T = 0$. However, one can study $m_b \neq m_f$ mass deformations of $\mathcal{N} = 4$ SYM at finite temperature.

$\Longrightarrow$To study holographic duality in full details, we need the full ten-dimensional background of type IIB supergravity, i.e, we need the lift of 5-dimensional gauged SUGRA solutions. This will be obvious when we discuss jet quenching in $\mathcal{N}=2^*$.

Such a lift was constructed in J.Liu,AB. Specifically, for any 5d solution, the 5d background:

$$ds_5^2 = g_{\mu\nu} dx^\mu dx^\nu \,, \qquad \text{plus} \qquad \{\alpha, \chi\}$$

is uplifted to a solution of 10d type IIB supergravity:

$$ds_{10(E)}^2 = \Omega^2 ds_5^2 + \Omega^2 \frac{4}{\rho^2} \left[\frac{1}{c} d\theta^2 + \rho^6 \cos^2(\theta) \left(\frac{\sigma_1^2}{c X_2} + \frac{\sigma_2^2 + \sigma_3^2}{X_1} \right) + \sin^2(\theta) \frac{1}{X_2} d\phi^2 \right]$$

$$\Omega^2 = \frac{(c X_1 X_2)^{1/4}}{\rho} \,, \qquad X_1 = \cos^2\theta + c(r)\rho^6 \sin^2\theta \,, \qquad X_2 = c\cos^2\theta + \rho^6 \sin^2\theta$$

with

$$c \equiv \cosh 2\chi \,,$$

plus dilaton-axion, various 3-form fluxes, various 5-form fluxes.

Thermodynamics of $\mathcal{N}=2^*$ for (non-)susy mass-deformations (with J.Liu,P.Kerner,...)

Consider metric ansatz:

$$ds_5^2 = -c_1^2(r)\, dt^2 + c_2^2(r)\ \left(dx_1^2 + dx_2^2 + dx_3^2\right) + dr^2$$

Introducing a new radial coordinate

$$x \equiv 1 - \frac{c_1}{c_2}\,,$$

with $x \longrightarrow 0_+$ being the boundary and $x \longrightarrow 1_-$ being the horizon, we find:

$$c_2'' + 4c_2\,(\alpha')^2 - \frac{1}{x-1}c_2' - \frac{5}{c_2}(c_2')^2 + \frac{4}{3}c_2\,(\chi')^2 = 0$$

$$\alpha''+\frac{1}{x-1}\,\alpha'-\frac{\frac{\partial\mathcal{P}}{\partial\alpha}}{12\,\mathcal{P}c_2^2(x-1)}\left[(x-1)\left(6(\alpha')^2+2(\chi')^2\right)c_2^2-3c_2'c_2-6(c_2')^2(x-1)\right]=0$$

$$\chi''+\frac{1}{x-1}\,\chi'-\frac{\frac{\partial\mathcal{P}}{\partial\chi}}{4\,\mathcal{P}c_2^2(x-1)}\left[(x-1)\left(6(\alpha')^2+2(\chi')^2\right)c_2^2-3c_2'c_2-6(c_2')^2(x-1)\right]=0$$

We look for a solution to above subject to the following (fixed) boundary conditions:

$\Longrightarrow$ near the boundary, $x \propto r^{-4} \to 0_+$

$$\left\{c_2(x), \alpha(x), \chi(x)\right\} \to \left\{x^{-1/4}, \qquad \frac{m_b^2}{T^2}\, x^{1/2} \ln x, \qquad \frac{m_f}{T}\, x^{1/4}\right\}$$

of course, we need a precise coefficients here relating the non-normalizable components of the sugra scalars to the gauge theory masses

$\Longrightarrow$ near the horizon, $x \to 1_-$ (to have a regular, non-singular Schwartzchild horizon)

$$\left\{c_2(x), \alpha(x), \chi(x)\right\} \to \left\{\text{constant}\,, \qquad \text{constant}\,, \qquad \text{constant}\right\}$$

System of above equations can be solved analytically when $\frac{m_b}{T} \ll 1$ and $\frac{m_f}{T} \ll 1$ With the help of the holographic renormalization (in this model AB) we can independently compute the free energy density $\mathcal{F} = -P$, the energy density $\mathcal{E}$, and the entropy density s of the resulting black brane solution:

$$-\mathcal{F} = P = \frac{1}{8}\pi^2 N^2 T^4 \left[1 - \frac{192}{\pi^2}\ \ln(\pi T)\, \delta_1^2 - \frac{8}{\pi}\, \delta_2^2\right]$$

$$\mathcal{E} = \frac{3}{8}\pi^2 N^2 T^4 \left[1 + \frac{64}{\pi^2}\ (\ln(\pi T) - 1)\ \delta_1^2 - \frac{8}{3\pi}\, \delta_2^2\right]$$

$$s = \frac{1}{2}\pi^2 N^2 T^3 \left(1 - \frac{48}{\pi^2}\, \delta_1^2 - \frac{4}{\pi}\, \delta_2^2\right)$$

with

$$\delta_1 = -\frac{1}{24\pi}\ \left(\frac{m_b}{T}\right)^2, \qquad \delta_2 = \frac{\left[\Gamma\left(\frac{3}{4}\right)\right]^2}{2\pi^{3/2}}\ \frac{m_f}{T}$$

A highly nontrivial consistency test on the analysis, as well as on the identification of gauge theory/supergravity parameters are the basic thermodynamics identities:

$$\mathcal{F} = \mathcal{E} - sT$$

$$d\mathcal{E} = T ds$$

$\Longrightarrow$ For finite (not small) m_b/T and m_f/T we need to do numerical analysis. However, we always check the consistency of the thermodynamic relations. In our numerics

$$\frac{d\mathcal{E} - T ds}{d\mathcal{E}} \sim 10^{-3}$$

The phase diagram of the model depends on

$$\Delta \equiv \frac{m_f^2}{m_b^2} :$$

- when $\Delta \geq 1$ there is no phase transition in the system;
- when $\Delta < 1$ there is a critical point in the system with the divergent specific heat. The corresponding critical exponent is $\alpha = 0.5$:

$$c_V \sim |1 - T_c/T|^{-\alpha}$$

where $T_c = T_c(\Delta)$.

For concreteness we discuss below 2 cases:

(a) $\Delta = 1$ ('susy' flows at finite temperature)

(b) $\Delta = 0$ ('bosonic' flows at finite temperature)

Before we discuss the flows, recall the lattice data for the QCD:

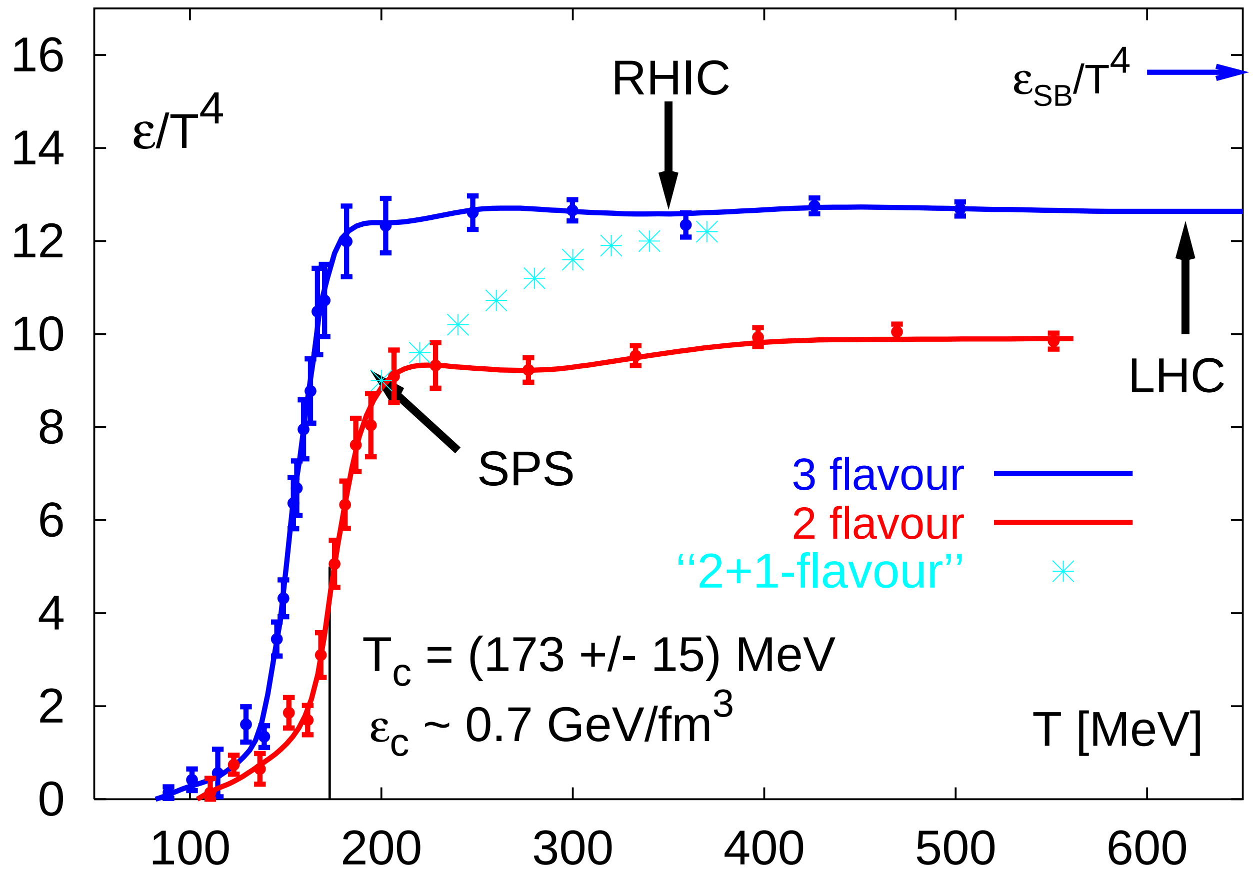

Figure 1: QCD thermodynamics from lattice; F.Karsch and E.Laermann, hep-lat/0305025.

- RHIC QGP is strongly coupled because equilibrium plasma temperature is roughly the QCD deconfinement temperature,

$$T_{plasma} \sim T_{deconfinement} \sim \Lambda_{QCD}$$

- Thus scale invariance is strongly broken and it is not clear why conformal $\mathcal{N} = 4$ plasma or near-conformal plasma thermodynamics/hydrodynamics should be relevant...

Surprisingly...

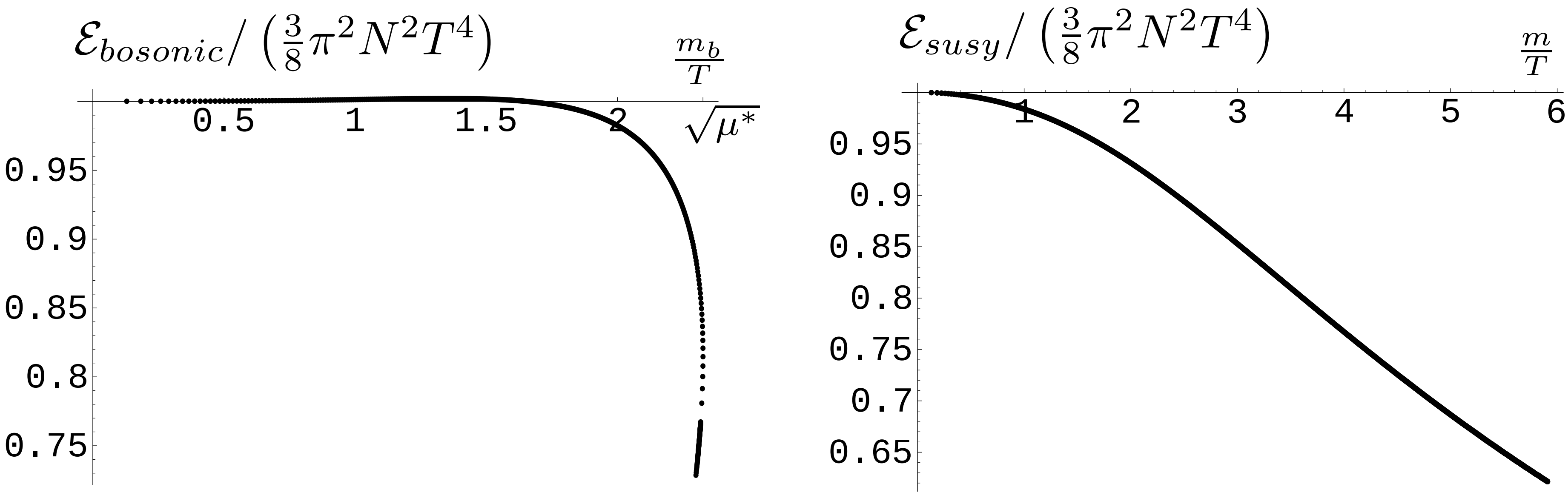

Figure 2: Equation of state of the mass deformed $\mathcal{N} = 4$ gauge theory plasma. At $T \sim m$ the deviation from the conformal thermodynamics is $\sim 2\%$. For the ideal gas approximation the deviation is about 40%. (S.Deakin, P.Kerner, J.Liu, AB, hep-th/0701142.)

$\Longrightarrow \mathcal{N} = 2^*$ model appears to share a 'thermodynamic plateau' with QCD!

Bulk viscosity of $\mathcal{N}=2^*$ for (non)-susy mass deformations

$\Longrightarrow$ How does sound propagates in viscous fluids?

Let $u^\mu = (u^0, u^i)$ — fluid 4-velocity. Introduce a proper (rest) frame for the fluid element

$$u^0 = 1, \qquad u^i = 0, \qquad , \qquad [\, \partial_\mu u^\nu \neq 0 \qquad \mathrm{off-equilibrium}\,]$$

$$T_{\mu\nu} = \Big\{ (P+\epsilon) u_\mu u_\nu + P\eta_{\mu\nu} \Big\} \qquad\qquad + \Big\{ \tau_{\mu\nu} \Big\}$$

$\Uparrow$ equilibrium stress tensor

$\Uparrow$ stress tensor due to velocity gradients

Definition of the rest frame: $\tau_{00}, \tau_{0i} = 0 \qquad \Rightarrow$

$$T_{00} = \epsilon \qquad ; \qquad T_{0i} = 0$$

“Constitutive” relation for remaining components:

$$\tau_{ij} = -\zeta \left\{ \delta_{ij}\, \partial_k u^k \right\} - \eta \left\{ \partial_i u_j + \partial_j u_i - \frac{2}{3} \delta_{ij}\, \partial_k u^u \right\}$$

ζ — couples to the trace of the velocity gradients — bulk viscosity

η — couples to the traceless part of the velocity gradients — shear viscosity

Stress-energy conservation:

$$\partial_0 \tilde{T}^{00} + \partial_i T^{0i} = 0 \qquad ; \qquad \partial_0 T^{0i} + \partial_j \tilde{T}^{ij} = 0$$

where $\tilde{T}^{00} \equiv T^{00} - \epsilon$, and

$$\tilde{T}^{ij} \equiv T^{ij} - P\delta^{ij} = -\frac{1}{\epsilon + P} \left[\eta \left(\partial^i T^{0j} + \partial^j T^{0i} - \frac{2}{3} \delta^{ij} \partial_k T^{0k} \right) + \zeta\, \delta^{ij} \partial_k T^{0k} \right]$$

We can study on-shell fluctuations (eigenmodes) of the above equations. Here we have two types of eigenmodes:

a: the shear mode (transverse fluctuations of the momentum density T^{0i})

$$\omega = -\frac{i\eta}{\epsilon + P}\, q^2 = -i\, \frac{\eta}{Ts}\, q^2$$

where we used $\epsilon + P = Ts$

b: sound mode (simultaneous fluctuations of the energy density $\tilde{T}^{00}$ and longitudinal component of T^{0i})

$$\omega = c_s\, q - \frac{i}{2}\, \frac{4}{3}\frac{\eta}{Ts}\, \left[1 + \frac{3\zeta}{4\eta}\right]\, q^2$$

c_s— the speed of sound

η, ζ— shear and bulk viscosity

$\Longrightarrow$ As we are interested in the bulk viscosity, we will concentrate on the “sound mode”

Kovtun+Starinets showed that the two hydrodynamic modes of the fluid can be identified with the lowest quasinormal modes of the holographic dual black brane geometry.

Definition: A quasinormal mode is an on-shell fluctuation of the black-brane background geometry subject to the following boundary conditions:

- an incoming-wave at the horizon
- have vanishing coefficients for all the non-normalizable modes of gravitational fields

$\Longrightarrow$ On the string theory/supergravity side the 'shear' quasinormal mode is simple — only the transverse traceless metric fluctuations get excited; the 'sound' quasinormal mode is technically much more involved, as the trace of the metric fluctuations would excite all the other matter fields — $\{\alpha, \chi\}$ sugra scalars of the $\mathcal{N} = 2^*$ holographic dual

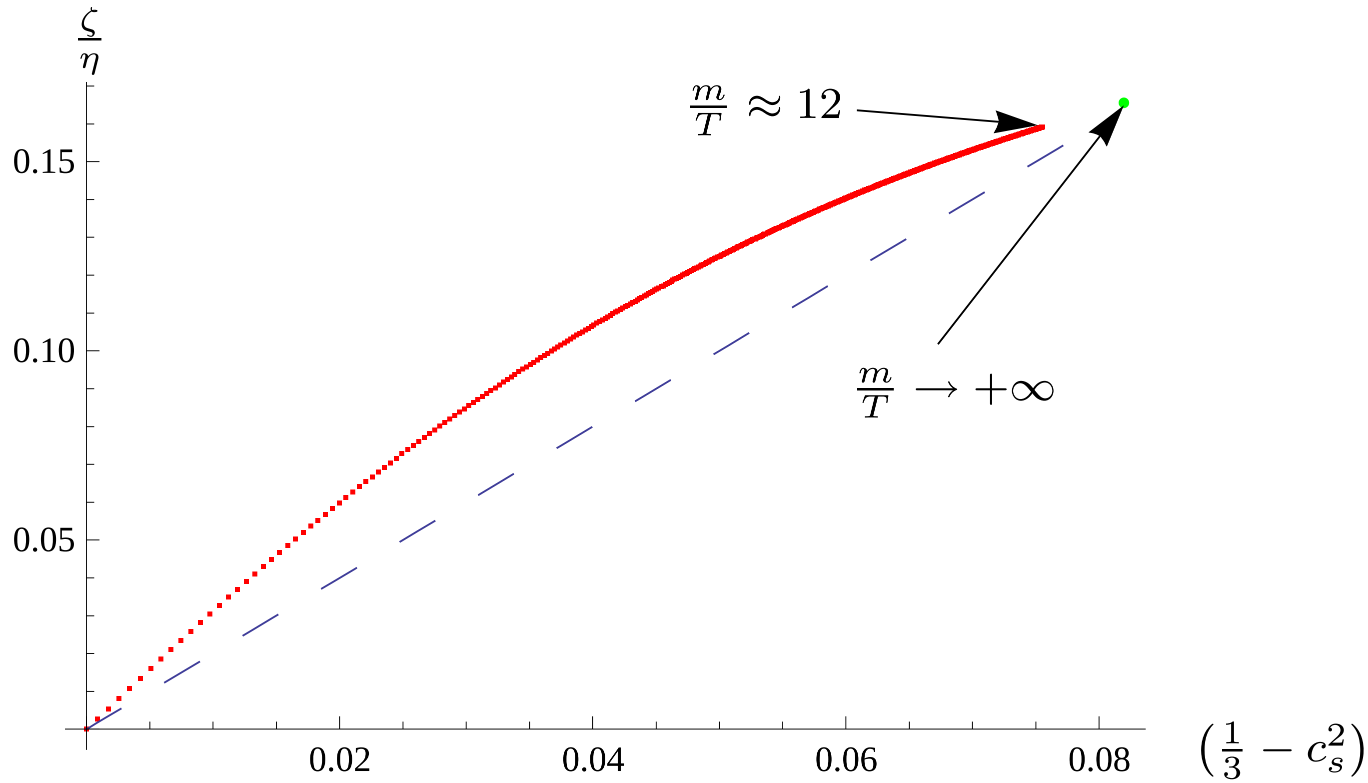

Figure 3: Ratio of viscosities $\frac{\zeta}{\eta}$ versus the speed of sound in $\mathcal{N}=2^*$ gauge theory plasma with "supersymmetric" mass deformation parameters $m_b = m_f = m$. The dashed line represents the bulk viscosity inequality $\frac{\zeta}{\eta} \ge 2\left(\frac{1}{3}-c_s^2\right)$. We computed the bulk viscosity up to $m/T \approx 12$. A single point represents extrapolation of the speed of sound and the viscosity ratio to $T \to +0$.

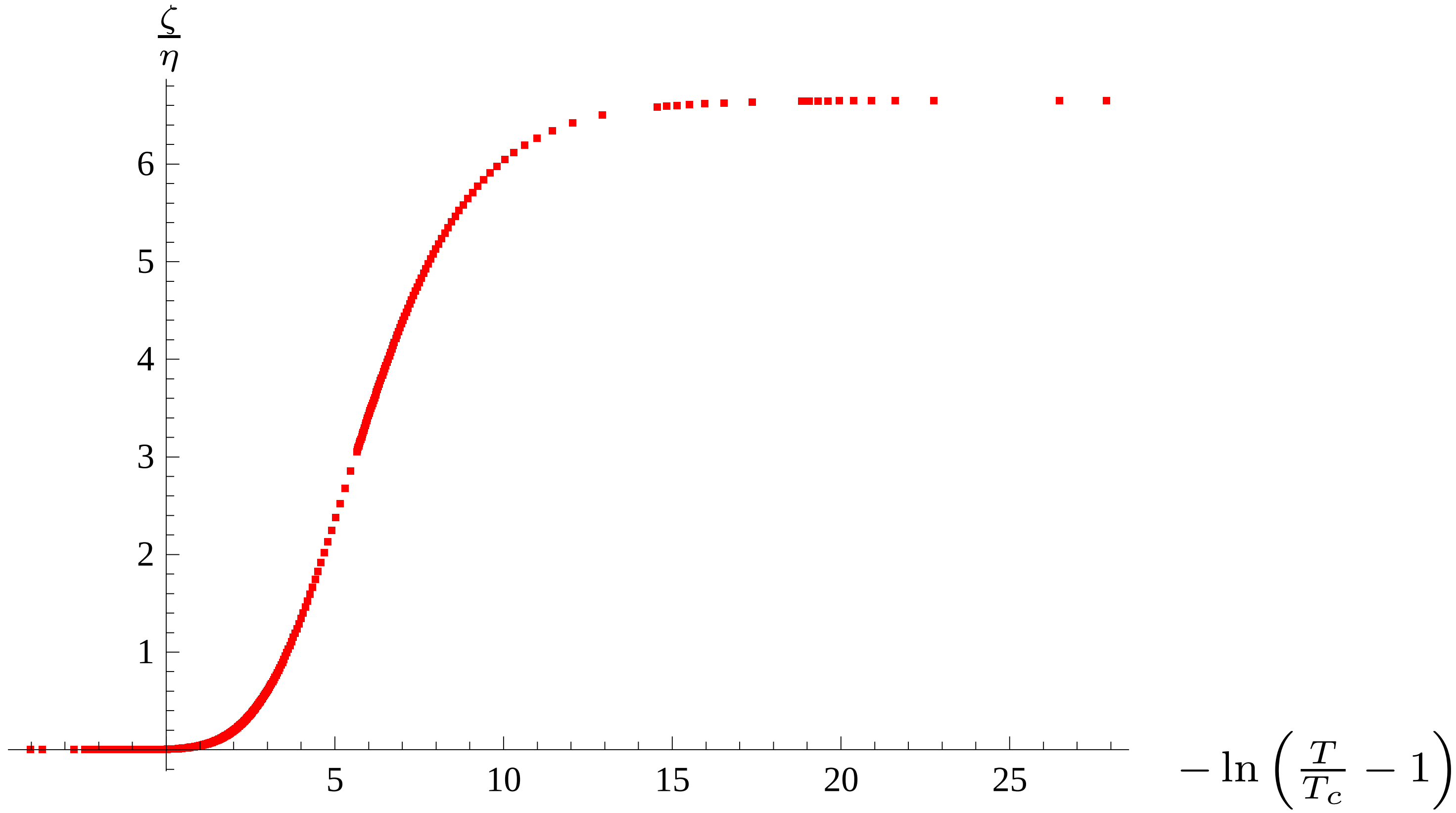

Figure 4: Ratio of viscosities $\frac{\zeta}{\eta}$ in $\mathcal{N}=2^*$ gauge theory plasma with zero fermionic mass deformation parameter $m_f=0$.

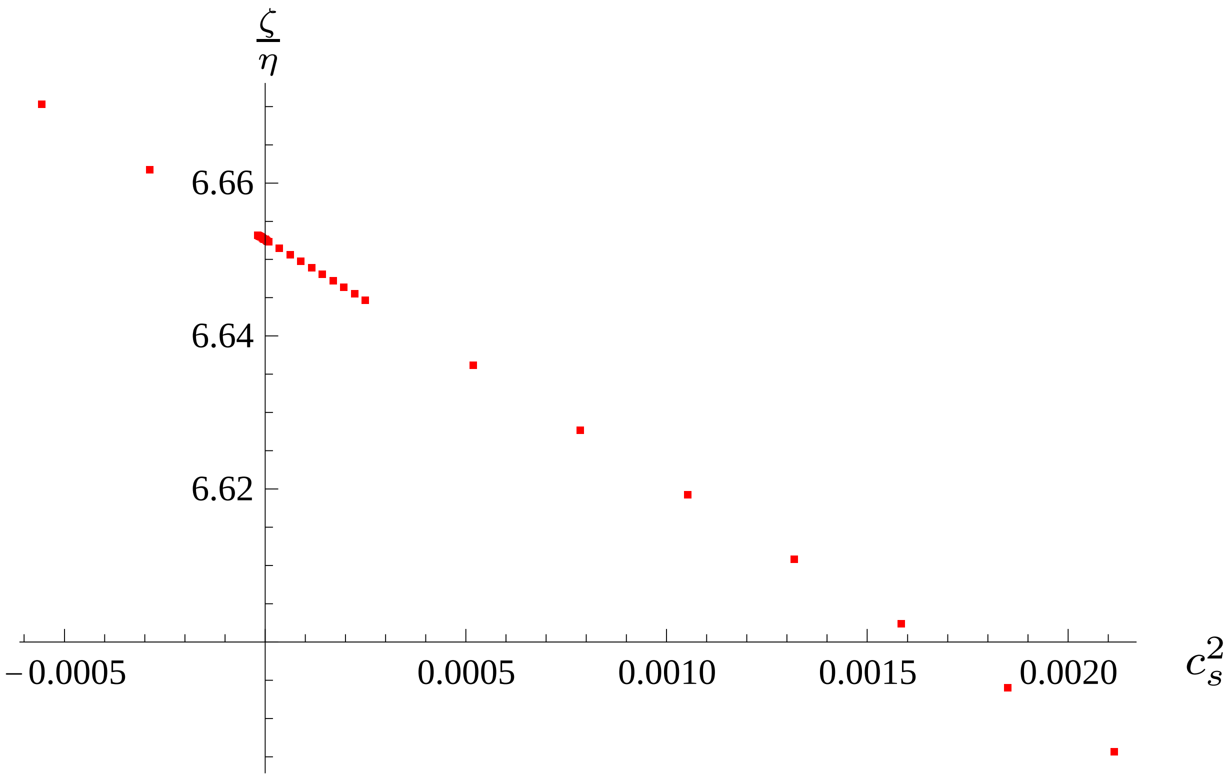

Figure 5: Ratio of viscosities $\frac{\zeta}{\eta}$ in $\mathcal{N} = 2^*$ gauge theory plasma near the critical point. Note that the critical point corresponds to $c_s^2 = 0$.

$\Longrightarrow$ Notice that the bulk viscosity is finite at the mean-field-theory critical point; the value favorably compares with Meyer's lattice simulations.

Estimates for the viscosity of QGP at RHIC. It is tempting to use the $\mathcal{N}=2^*$ strongly coupled gauge theory plasma results to estimate the bulk viscosity of QGP produced at RHIC. For c_s^2 in the range $0.27-0.31$, as in QCD at $T=1.5T_{deconfinement}$ we find

$$\frac{\zeta}{\eta}\bigg|_{m_f=0}\approx 0.17-0.61\,,\qquad \frac{\zeta}{\eta}\bigg|_{m_b=m_f=m}\approx 0.07-0.15\,. \tag{1}$$

Since RHIC produces QGP close to its criticality, we believe that $m_f=0$ $\mathcal{N}=2^*$ gauge theory model would reflect physics more accurately.

”$\mathcal{N} = 2^*$-based” thermometer for RHIC

Rajagopal et.al proposed to parametrized the quenching of the partonic jets in QGP in terms of the ”jet quenching parameter” $\hat{q}$, defined as follows:
consider a light-like Wilson loop C with large extension L^- in x^- direction and small extension L in transverse direction; then

$$< W^A(C) >= \exp\left(-\frac{1}{4}\hat{q}L^-L^2 + \mathcal{O}(L^4)\right)$$

$\Longrightarrow$ On the dual supergravity side, the computation of $\hat{q}$ reduces to finding the minimal string world-sheet, which has a gauge theory Wilson loop boundary at the boundary of the AdS$_5$.

$\Longrightarrow$ The computations must be performed in 10d dual geometry; however for $\mathcal{N} = 4$ SYM plasma the $SO(6)$ R-symmetry implies that the minimal string world-sheet can be localized at any point on S^5.

$\Longrightarrow$ Things are more complicated in $\mathcal{N} = 2^*$ plasma, where the R-symmetry is broken:

$$SO(6) \to SU(2) \times U(1)$$

Indeed, we find that the minimal world-sheet is localized on different points of the 'squashed' S^5, depending on the values of sugra scalars $\{\alpha, \chi\}$ (or $\{m_b/T, m_f/T\}$ in the gauge theory language):

$$\theta_{min} = \begin{cases} 0 & \text{if } c \leq \rho^6 \\ \frac{\pi}{2} & \text{if } c > \rho^6 \end{cases}$$

Again, in the limit of $\frac{m_b}{T} \ll 1$, $\frac{m_f}{T} \ll 1$ we can find analytically

$$\hat{q} = \frac{\pi^2}{a}\sqrt{\lambda}T^3\left(1 - \kappa_1\delta_1 - \left(\kappa_2 + \frac{4}{\pi}\right)\delta_2^2 + \mathcal{O}\left(\delta_1^2, \delta_1\delta_2^2, \delta_2^4\right)\right), \; a \equiv \frac{\sqrt{\pi}\Gamma\left(\frac{5}{4}\right)}{\Gamma\left(\frac{3}{4}\right)}$$

where

$$\kappa_1 \approx -1.13036\,, \qquad \kappa_2 \approx -0.37152$$

Recall that:

$$\delta_1 = -\frac{1}{24\pi}\,\left(\frac{m_b}{T}\right)^2, \qquad \delta_2 = \frac{\left[\Gamma\left(\frac{3}{4}\right)\right]^2}{2\pi^{3/2}}\,\frac{m_f}{T}$$

$\Longrightarrow$ Notice that for a fixed temperature T the quenching of partonic jets in $\mathcal{N} = 2^*$ plasma is *less* than the quenching of partonic jets in $\mathcal{N} = 4$ plasma.

Thermometer profiles:

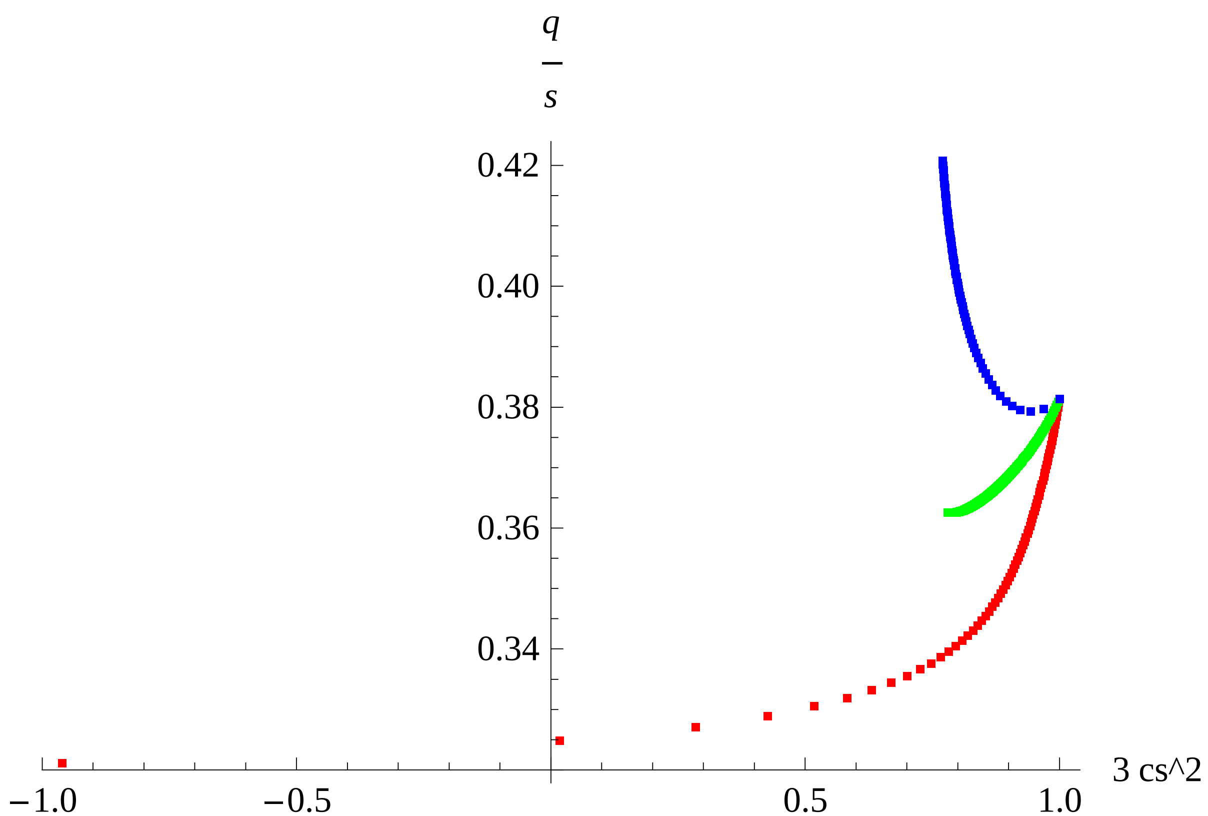

Figure 6: The ratio $\frac{\hat{q}}{s}$ as a function of the speed of sound for various values of $\Delta = \frac{m_f^2}{m_b^2}$. Green points: $\Delta = 1$; blue points: $\Delta = 1.5$; red points: $\Delta = 0.5$.

$\Longrightarrow$ Unfortunately, our thermometer is not universal!

Instructions for a thermometer usage:

- pick-up the appropriate ”thermometer” profile — we argued previously that the $\Delta < 1$ profiles might be more appropriate for the QGP at RHIC;
- given c_s, use QCD lattice data to extract $T/T^{QCD}_{deconfinement}$ and s^{QCD}
- determine QGP jet quenching as

$$\hat{q}^{QGP} = \left(\mathrm{profile}\right) \times s^{QCD}$$

Conclusions and future directions

I hope I convinced you that $\mathcal{N} = 2^*$ model of gauge theory/string theory duality is a valuable tool (toy model) to study properties of QGP at RHIC, and hopefully LHC

I think the most important and interesting problem for the future is understanding the hadronization of the RHIC plasma ball. I believe $\mathcal{N} = 2^*$ model can be valuable here as well, given that it has a critical temperature.